LECTURE-NOTES

ON THE

THEORY OF ELECTRICAL MEASUREMENTS.

PREPARED FOR THE THIRD-YEAR CLASSES OF THE COOPER UNION NIGHT-SCHOOL OF SCIENCE.

BY

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PREFACE.

It is the purpose of these Notes to furnish the student the topics treated in the lectures; to give in full such facts, data, and courses of reasoning as are not readily mastered by the student in the course of the lecture; in short, to aid the student in those matters which he is likely to find most difficult, or which he may fail at the moment to fully grasp. It is not, however, intended to relieve the student of the necessity of taking his own notes of the lectures as they are delivered, and every student is expected to take such notes, especially as to the experiments and the lessons which they teach. Unless the student writes short-hand, the notes he can take during a lecture must necessarily be brief, and he should aim to record suggestive phrases to be afterward more fully elaborated.

The student is advised of the great importance, as soon as possible after the lecture, of going over the ground and filling in his notes, thereby clearing up

subjects that may have appeared obscure, and fixing his knowledge of the topics treated. On no account should the student fail to make this review before the time for the next lecture.

It is the purpose of this course of lectures to teach the fundamental principles of all electrical measurements, rather than to describe in detail methods employed in particular cases. To this end the definitions and relations of electrical quantities have been discussed at considerable length, and the derivation of the electrical units has been fully treated. It is believed that, with this thorough foundation in the theory of electrical measurements, the student will be better equipped for practical work with a knowledge of typical methods such as are treated in these lectures, than he could possibly be by the most extended empirical instruction in special methods without such foundation.

CONTENTS.

NOTES

UPON

ELECTRICAL MEASUREMENTS.

THE C.G.S. SYSTEM OF UNITS.

To measure any quantity is to compare it with another quantity of the *same kind* that has been chosen as a unit.

It was the practice originally to choose units arbitrarily, without regard to the relation they might bear to other units already chosen. Thus the gallon, a unit of capacity, is 231 cubic inches. It would have been far more convenient if it had been 100 or 1000 cubic inches.

When electrical science had reached the stage where units of measurement were necessary, units were at first arbitrarily chosen, but it was very early recognized that such a course would lead to a multitude of troublesome factors for reducing to other

units already fixed, and, fortunately, the matter was taken up and a consistent system of units devised before the arbitrary units had acquired a strong foothold.

The system so devised is known as the centimetre-gramme-second, or the c.g.s., system of units. In order that it may be fully understood, it will be necessary to consider briefly the basis upon which the system rests.

It has been said that all physical phenomena are the result of matter and motion. Any physical measurement may, therefore, be effected by measuring the mass of matter, and the motion involved. But motion implies two elements—space or length, and time. The measurement of any physical phenomenon, therefore, involves the measurement of three independent quantities, and for the measurement of these quantities three independent units must be arbitrarily chosen. These are called fundamental units, and from them all other units may be derived.

Fundamental Units.—The fundamental units of the c.g.s. system are: for the unit of length, the centimetre, which is the one-hundredth part of the length of a certain platinum bar deposited in the archives of France and declared by government enactment to be a metre; for the unit of mass the gramme, which is the one-thousandth part of a cube of platinum

deposited in the archives of France and declared by government enactment to be a kilogramme; for the unit of time, the second, which is the 86400th part of the mean solar day.

Characteristics of Matter.—The distinctive characteristic of matter is its persistence in whatever state of rest or motion it may happen to have, and the *resistance* which it offers to any attempt to change that state. This property is called *inertia.*

The resistance which a body offers to change of state is, other things being equal, proportional to its *mass.*

Anything that changes the state of a body with respect to rest or motion is called *force.*

Motion.—Motion may be *uniform* or *varied.* Rate of motion is called velocity. In a uniform motion, velocity is measured by the space passed over in unit time. In varied motion, the velocity at any instant of time is measured by the space which the body would pass over in the next second if, from that instant, the velocity were uniform.

The unit velocity is the velocity of a body which, moving with a uniform motion in a straight line, describes unit space in unit time.

Rate of change of velocity is called *acceleration.* Acceleration may be constant or varied.

When constant, it is measured by the change in velocity which occurs in unit time. When varied, its value at any instant is measured by the change in

velocity that would occur in unit time if, from that instant, the acceleration were constant.

Unit acceleration is the acceleration of a body whose velocity, changing at a constant rate, changes by the amount of one unit velocity in the unit time.

A constant velocity is expressed by the ratio space to time:

$$v = \frac{s}{t}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

This expression is also true for the instantaneous velocity at a given instant of time in varied motion, if s represent a space described at that instant in a time t so short that during that time the velocity may be considered constant.

In the same way, a constant acceleration is expressed by the ratio velocity to time:

$$a = \frac{v}{t}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

v representing the change of velocity in the time t. If v and t be taken small enough, formula (2) gives the instantaneous acceleration when acceleration is variable, just as (1) gives the instantaneous velocity in varied motion.

Uniformly Varied Motion, or Motion in which Acceleration is Constant.—Let v represent the initial velocity. Then velocity at end of time t is

$$v = v_1 + at. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Since the velocities in this case form a series in arithmetical progression, the mean velocity is the half-sum of the initial and final velocities, or

$$\frac{v_1 + v}{2} = \frac{v_1 + v_1 + at}{2} = v_1 + \frac{at}{2},$$

and the space described in time t is

$$s = \frac{v_1 + v}{2} t, \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

or

$$s = v_1 t + \tfrac{1}{2} a t^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Substitute in (4) the value of t from (3):

$$s = \frac{v^2 - v_1^2}{2a}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

If the motion start from rest, $v_1 = 0$, and the above formulæ become

$$v = at; \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$s = \tfrac{1}{2} vt; \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$s = \tfrac{1}{2} at^2; \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

$$s = \frac{v^2}{2a}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Force.—Since force is the assumed cause of change of motion, it is proper to take the change produced

in unit mass in unit time, that is, the *rate* of change of motion, or acceleration, as a measure of the force producing it. But the force required to produce a given change is proportional to the *mass*. Hence the force required to produce a given acceleration in any mass is proportional to the product of the mass by acceleration.

Since we are free to choose a unit of force, we may make the force

$$F = ma, \quad . \quad . \quad . \quad . \quad . \quad (11)$$

m being any mass, and a the acceleration produced in that mass by the force F. We may now define the c.g.s. unit force as that force which may produce in a mass of one gramme the c.g.s. unit acceleration. It is called a *dyne*.

Work and Energy.—The physical idea of work is resistance *overcome* through space. Assuming a force to produce motion in its own direction, work is measured by the product of that force by the space through which it acts. Hence the following equation:

$$W = Fs. \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The c.g.s. unit work is the work performed by a force of one dyne acting through a space of one centimetre in the direction of the force. It is called an *erg*.

When the resistance overcome is inertia, the force

overcoming it equals ma; see (11). If the force continue to act until the velocity suffers a change equal to v, then the space through which it acts is (10), $s = \frac{v^2}{2a}$. Substituting these values of F and s in (12),

$$W = \tfrac{1}{2}mv^2. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (13)$$

That is, the work which must be done to impart to the mass m the velocity v equals half the product of the mass by the square of its velocity. Conversely, if a mass m, moving with a velocity v, be brought to rest, it will *do* work represented by the same product.

Energy is capacity for doing work. Thus the mass of the last paragraph has a capacity for doing work equal to $\frac{1}{2}mv^2$. This is the energy it possesses because of its motion.

A body may also possess energy because of its *position of advantage* with respect to some force. The energy which a body possesses because of its motion is called *kinetic* energy. That which it possesses because of its position is called *potential* energy.

Energy is measured in the same units as work. In describing the measurement of work it was assumed that the displacement of the body was in the line of direction of the force, but this may not be the case. The body may be constrained to move in a path whose direction makes an angle with the line of direction of the force. In this case, to obtain the *effective*

force along the path, the force must be resolved into two components, one in, and the other at right angles to, the path. Evidently the component lying in the path is the only component having any effect to produce motion along the path, and the work done is measured by the product of this component into the space through which the point of application of force moves. The component along the path is the *projection* of the force upon the path. If α be the angle between the path and the line of direction of the force, the effective component along the path is $F \cos \alpha$.

Practical Units.—The erg is a very small amount of work. For practical purposes a larger unit, equal to ten million (10^7) ergs, is employed, and is called a *joule*.

The *watt* is a unit *rate* of working. It is work performed at the rate of one joule per second.

Gravitation Units.—Before the adoption of the simply related units of the c.g.s. system certain arbitrary units of force and work were in use, and are still largely used in engineering practice. Those most in use are: a force equal to the weight of a pound of matter, called also a pound, and a force equal to the weight of a kilogramme at Paris, called also a kilogramme. It is unfortunate that the name pound should have been used for two such totally distinct quantities.

From these units of force are derived the units of

work, the *foot-pound,* being the work done by a force of one pound acting through a space of one foot, and the *kilogramme-metre,* being the work done by a force of one kilogramme acting through a space of one metre; also the units rate of working, the *horse-power,* being work performed at the rate of 550 foot-pounds per second, and the *cheval-vapeur,* work performed at the rate of 75 kilogramme-metres per second.

Heat Units.—Since heat is a form of energy, it is important to define here the heat units and their relation to the other units of work and energy.

The *British thermal unit* is the heat required to raise the temperature of one pound of water from 32° to 33° Fahrenheit.

The *pound-degree centigrade* is the heat required to raise the temperature of one pound of water from zero to 1° centigrade.

The *calorie* is the heat required to raise the temperature of one kilogramme of water from zero to 1° centigrade.

The *lesser calorie* is the heat required to raise the temperature of one gramme of water from zero to 1° centigrade.

Below are given the relations between these various units.

UNITS OF FORCE.

Kilogramme	= 981000 dynes.
Pound	= 444972 "

UNITS OF WORK OR ENERGY.

Kilogramme-metre	$= 9.81 \times 10^7$ ergs.
"	$= 9.81$ joules.
Foot-pound	$= 1.356 \times 10^7$ ergs.
"	$= 1.356$ joules.

UNITS RATE OF WORKING.

Cheval-vapeur	= 736 watts.
Horse-power	= 746 "

UNITS OF HEAT.

Calorie	= 426 kgm.-metres.
"	= 4160 joules.
British thermal unit	= 778 foot-pounds.
" " "	= 1055 joules.
Pound-degree centigrade	= 1400 foot-pounds.
" "	= 1898 joules.

Joule = .00024 calorie
= .000948 B. T. U.
= .000527 pound-deg. C.

PROBLEMS.

(1) A body is projected upward with a velocity of 5000 cm. How high will it rise?

(2) A rifle-barrel is 75 cm. long. A rifle-ball of 15 grms. leaves the rifle with a velocity of 60000 cm. per second. Assuming a uniform acceleration, for how

long a time was the bullet in the rifle? What was the force of the powder in dynes? in pounds?

(3) If the bullet of the last example were stopped in a space of 10 cm. by a uniform resistance, what is that resistance?

(4) What energy—ergs, joules, foot-pounds—did the bullet possess on leaving the rifle?

(5) How much heat—calories, B.T.U.—can be generated in one hour by one horse-power?

THE MAGNETIC FIELD.

Definition of a magnet.

Natural and artificial magnets.

Bar magnets, horseshoe magnets.

Distribution of magnetic force.

Ends where force is manifested called *poles*.

The two poles are not alike.

Any magnet suspended so as to be free to swing in a horizontal plane settles with one pole toward the north and the other toward the south. One pole cannot exist without the other.

Pole pointing north is in English writings called the *north pole*.

Mutual action of poles.

Force extends all around the magnet and to a great distance.

Space around a magnet where its forces are manifested is called the *magnetic field*.

Direction and intensity of forces in different parts of the field vary greatly.

A curved line so drawn in the field that at each

point of this line the line of direction of the magnetic force at that point is tangent to it is called a *line of force*.

The direction of the force along the line is assumed to be the direction in which a free *north* pole would move in obedience to that force.

Lines of force indicated by iron-filings and by a small needle.

Lines of force can never cross each other, for, if they did, that would mean two directions of the magnetic force at the point of crossing. This is impossible.

Strength of Magnetic Poles.—Forces exerted by different poles vary greatly.

Force exerted by the *same* pole at different distances varies greatly.

Theory indicates and careful measurements demonstrate that the magnetic force exerted by one magnetic pole upon another varies inversely as the square of the distance.

The strength of a magnetic pole is assumed, other things being equal, to be proportional to the force exerted by it.

Unit Magnetic Pole.—Since the action between two magnetic poles is mutual, the force exerted by one pole upon another must be proportional to the product of the two pole strengths. Hence, as we are

free to choose the unit strength of pole, we may put

$$F = \frac{pp}{d^2}. \quad . \quad . \quad . \quad . \quad . \quad (14)$$

The c.g.s. unit strength of pole is, then, a pole of such strength that it repels another equal and similar pole at a distance of one centimetre with a force of one dyne.

Intensity of Magnetic Field.—It is often convenient to express magnetic forces in terms of the intensity of the magnetic field. The intensity of the magnetic field at any point is assumed to be proportional to the force exerted upon a pole of given strength placed at that point. Hence we may put

$$F = Hp, \quad . \quad . \quad . \quad . \quad . \quad (15)$$

where H is the intensity of the field.

The c.g.s. unit intensity of field is now a field of such intensity that in it unit pole is actuated by a force of one dyne.

Since unit pole is also actuated by a force of one dyne at a distance of one centimetre from another unit pole, it follows that unit intensity of field is found at a distance of one centimetre from the unit pole.

These definitions have been given as though one pole alone could act to produce a magnetic field, but this is a condition that cannot be realized in practice.

It has already been stated that one pole could not exist alone. It is also true that each pole affects to a greater or less extent the intensity and direction of the lines of force in the magnetic field.

Graphical Representation of Magnetic Field.—Lines of force may not only be used to indicate the direction of the forces in the magnetic field, but they may be so drawn as to indicate the intensity also. For this purpose they are so drawn that the number of lines passing through a square centimetre at any point is equal to the number of units expressing the field intensity at that point.

It is customary to refer to the intensity of a magnetic field by saying it is a field having so many lines.

Fields of from 1000 to 16000 lines are met with in electric generators and motors.

The Earth's Magnetic Field.—The lines of force of the earth's field in this region lie in a vertical plane deviating by some 7° to the west of the geographical meridian, and are inclined at an angle of some 75° with the horizon.

The deviation from the true north is called the *declination* of the magnetic needle, and the deviation from the horizontal is called the *inclination* or *dip*. Since magnetic needles are usually free to swing only in a horizontal plane, they are affected only by the *horizontal component* of the earth's magnetic intensity. This is called the horizontal intensity of the earth's

magnetic field, and is represented by the symbol H. If α be the angle of dip, then

$$H = \text{total intensity} \times \cos \alpha.$$

Measurement of Field Intensity.—Suppose it is required to determine the horizontal intensity of the earth's magnetic field. A magnetic needle is suspended by a fibre of silk so as to be free to swing in a horizontal plane. If it be deviated slightly from the magnetic meridian and then left to itself, it will vibrate back and forth in a time which will be less the greater the force, and greater the greater the resistance offered by its inertia.

The resistance offered by the inertia of a body to a force producing rotary motion depends not only upon the mass of the body, but also upon the distribution of that mass with respect to the axis of rotation; that is, upon a factor of the body which is called the *moment of inertia.* This is a factor that may be determined by computation for some bodies of regular form, but for irregular bodies it must be determined by experiment. We will represent it by the symbol I.

Let NS in the figure represent the magnetic meridian, and A a magnetic needle deflected from the meridian plane. A force at each end of the needle, equal to Hp, constitutes a *couple* which causes the needle to vibrate. The effect of this couple to pro-

duce rotation is measured by the product of one of the forces by the distance between them, which, when the needle is at right angles to the meridian, is the length of the needle, hence by Hpl, where l is the length of the needle. pl, represented by the symbol

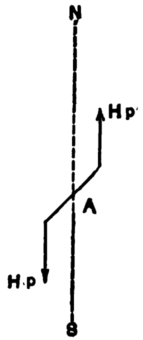

FIG. 1.

M, is called the *magnetic moment* of the needle. Then it can be shown that the time of a single vibration or oscillation is

$$t = \pi\sqrt{\frac{I}{HM}}. \quad . \quad . \quad . \quad . \quad (16)$$

If t and I are determined, HM may be found from formula (16).

But the quantity sought is H, and the above experiment gives the product HM.

To find H it is necessary to make another measurement to determine the ratio of H to M.

Let the magnet *A* of Fig. 1 be placed as in Fig. 2, at some distance to the east or west of a small needle

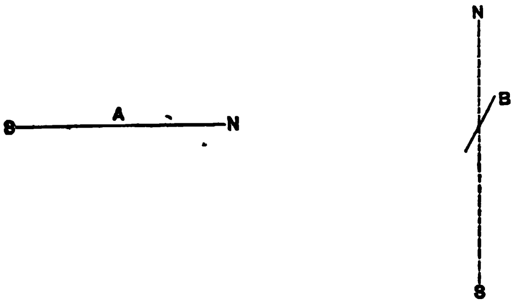

FIG. 2.

B whose deflection can be accurately measured. *A* is so placed that its axis prolonged passes through the centre of *B*. Then if *M* be the magnetic moment of *A*, and M_1 that of *B*, *r* being the distance between the centres, and θ the deflection of *B* produced by *A*, it can be shown that

$$\frac{2MM_1 \cos\theta}{r^3} = M_1 H \sin\theta;$$

$$\frac{M}{H} = \tfrac{1}{2}r^3 \tan\theta. \quad . \quad . \quad . \quad (17)$$

From (16) and (17) both *H* and *M* may be found.

The method here described for determining *H* is similar to that employed for determining the intensity of gravity by means of the pendulum.

Horizontal intensity of the earth's field in New York is about .17 c.g.s. units.

It has been suggested to adopt as a practical unit of field intensity the intensity of a uniform field in which there are 10^8 c.g.s. lines per cm.^{2}. This unit has been called a *gauss*. A field of 10000 c.g.s. units would be $\frac{1}{10}$ milligauss.

PROBLEMS.

(6) A magnetic needle vibrates 15 times per minute in the earth's magnetic field in New York. The same needle vibrates 16 times per minute in another location. What is the value of H in the second location?

(7) Two needles identical in dimensions and mass make, at the same place, one 50, the other 30 vibrations per minute. How do their strengths compare?

(8) If two needles known to be of the same strength make, at the same place, one 50, the other 30 vibrations, how is this accounted for? Explain the relation between the two.

(9) The needle A of Fig. 2 causes B to be deflected 15°. Another needle substituted for A causes a deflection of 20°. Required the relative strengths of the two needles.

(10) If in Fig. 2 the distance AB is 50 cm. and the needle B is deflected 15°, what is the relation between the magnetic moment of A and the field in which B is placed?

THE ELECTRIC CURRENT.

When the two terminals of an electric generator are joined by a conductor, something takes place which is called an electric flow.

The conductor is said to carry an *electric current*. This current is known only by its effects.

It heats the conductor.

It affects a magnetic needle near which the conductor may be placed. This shows that the current develops a magnetic field.

Field due to Current.—*Direction.*—Studying the effect of the current upon a needle, it is seen that the lines of force are concentric circles of which the wire is the axis.

Can be shown by means of iron-filings sprinkled on a glass plate.

The direction of the force in these lines may be determined from the following rule: *Suppose the current flowing from you. The lines then have the direction of the movement of the hands of a watch whose dial faces you.*

Intensity.—Let *AB*, Fig. 3, represent the direction of the horizontal component of the earth's magnetism.

Let a conductor carrying a current be placed parallel to *AB* and over or under the magnetic needle *NS*.

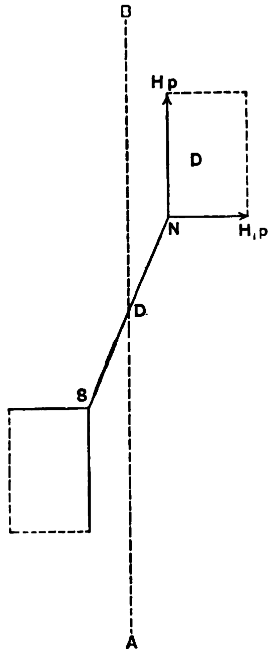

FIG. 3.

The lines of force due to the current are at right angles to *AB*, and forces in opposite directions, constituting a couple, will act upon the opposite poles of the needle.

These will cause a deflection which is opposed by the force of the earth's magnetism, and it is evident that the needle will be in equilibrium when its direction is that of the resultants of the two pairs of forces,

as represented in the figure. The force due to the earth's magnetism acting upon each pole of the needle is Hp. That due to the current is H_1p at right angles to Hp. If α be the angle of deflection, it is plain from the figure that

$$H_1p = Hp \tan \alpha,$$

or

$$H_1 = H \tan \alpha. \quad . \quad . \quad . \quad . \quad . \quad (18)$$

If the conductor, instead of remaining in the magnetic meridian, is turned with the needle and kept parallel to it, the deflecting force will always be at

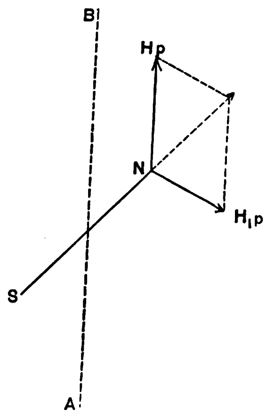

FIG. 4.

right angles to the needle as in Fig. 4, where only one force of each pair is shown. Here we have

$$H_1p = Hp \sin \alpha,$$

or

$$H_1 = H \sin \alpha. \quad . \quad . \quad . \quad . \quad (19)$$

It may be assumed as self-evident that if all parts of a conductor were at equal distances from a needle, the force exerted would be proportional to the length of the conductor.

It is shown experimentally that, other things being equal, the force exerted by a current upon a needle is proportional to the inverse square of the distance.

It is assumed that, other things being equal, *strength of current* is proportional to the force it exerts upon a magnetic pole. Hence we may put

$$F = \frac{CLp}{d^2}, \quad . \quad . \quad . \quad . \quad . \quad (20)$$

where C is the strength of current, L its length, d its distance from the pole whose strength is p.

We may now define the c.g.s. unit current as a current of such strength that, flowing in a conductor 1 cm. long, bent into an arc of a circle of 1 cm. radius, it will exert upon the c.g.s. unit magnetic pole at the centre of the circle a force of one dyne.

The c.g.s. unit current has no name. The unit used in practice is one tenth of the c.g.s. unit, and is called the *ampere*.

It is impossible practically to realize the conditions stated in the definition of the unit current, because

the conductors by which the current is brought to and carried away from the arc of unit length would themselves have some influence upon the needle. But if the conductor be bent into a complete circle of unit radius, the conductors leading to and from the circle may then be twisted together so that the currents flowing in them in opposite directions neutralize each other, and the effect produced is that of the circle alone. The length of such a circle being 2π, the field produced at the centre by unit current flowing in the conductor will be 2π.

A conductor forming a circle of radius r will, therefore, produce at its centre a magnetic field:

$$H_1 = \frac{C 2\pi r}{r^2} = \frac{2\pi}{r} C.$$

Tangent Galvanometer.—If the plane of such a circle coincide with the plane of the earth's magnetic meridian, and a short magnetic needle free to swing in a horizontal plane be placed at its centre, it will be in equilibrium (see (18)), when

$$\frac{2\pi}{r} C = H \tan \alpha. \quad . \quad . \quad . \quad . \quad (21)$$

Hence $$C = \frac{Hr}{2\pi} \tan \alpha. \quad . \quad . \quad . \quad . \quad (22)$$

If the conductor make n turns,

$$C = \frac{Hr}{2\pi n} \tan \alpha. \quad . \quad . \quad . \quad . \quad (23)$$

Note that the result is independent of the strength of the needle.

An instrument constructed to realize these conditions is called a tangent galvanometer because the current is proportional to the tangent of the angle of deflection.

The quantity $\frac{Hr}{2\pi n}$ is the *constant* of the galvanometer.

Disadvantages of this form of tangent galvanometer.

Helmholtz's Form of Tangent Galvanometer.—In this instrument there are two equal coils placed at a distance apart equal to their radius, and the needle is on the common axis midway between them.

The constant of this instrument is

$$.222\frac{Hr}{n}. \qquad . \quad . \quad . \quad . \quad . \quad . \qquad (24)$$

Current-measuring Instruments employing Artificial Fields.—The tangent galvanometer, depending for its indications upon the intensity of the earth's magnetic field, can only be used where this intensity is constant, or at least free from local disturbances. Furthermore, large currents cannot be accurately measured by this instrument because the fields produced by such currents are very large in comparison with the field of the earth. Instruments are therefore

constructed in which a strong artificial field produced by a permanent magnet is employed to direct the needle. Such instruments must be *calibrated* by direct comparison with some standard instrument.

Other Current-measuring Instruments.—Instead of employing a magnetic field as the directive force to oppose the force of the current, a spring may be employed.

Instead of making the magnetic needle movable, this may be fixed and the coil made the movable element.

Current-measuring instruments are often called *amperemeters* or *ammeters*.

Mutual Action of Currents.—Since every electric current produces a magnetic field, there must be a mutual action between any two currents placed near each other. The following rules apply to this mutual action: Parallel currents flowing in the same direction attract, flowing in opposite directions they repel.

Currents making an angle tend to become parallel and to flow in the same direction.

The following general law applies to all cases:

A current in any magnetic field tends to move in such a way as to increase the number of lines of force enclosing it.

Currents may be measured by means of their mutual action. For this purpose two coils are so connected that the same current traverses both and

the force due to their mutual action is measured. Evidently this force is proportional to the square of the current.

Electrodynamometers.

Electric balances.

Electrical Quantity.—It is sometimes necessary to consider the total quantity of electricity employed during a period of time. It is assumed that the unit current conveys the unit quantity per second. Hence the quantity conveyed in t seconds is

$$Q = Ct. \qquad . \quad . \quad . \quad . \quad . \qquad (25)$$

The practical unit quantity is the *coulomb*, which is the quantity conveyed by the ampere in one second.

PROBLEMS.

(11) Where $H = .17$, what is the constant of a tangent galvanometer having a coil of fifty turns 50 cm. diameter?

(12) If the needle of such an instrument is deflected 30°, what is the current in amperes? What quantity of electricity per hour is conveyed by such a current?

(13) If it be required to measure a current approximating 100 amperes, how large a coil of one turn would be required, the allowable deflection being 60°?

(14) What field intensity exists at the centre of a

circular coil of one turn one metre in diameter, carrying 1000 amperes?

(15) A tangent galvanometer has two coils, one 80 cm. diameter, ten turns, the other 100 cm. diameter, eight turns. Currents measured by the two coils respectively produce the same deflection. What is the relation between those currents?

POTENTIAL

AND

ELECTROMOTIVE FORCE.

Potential is a concept that was introduced into mechanics for the purpose of simplifying the study of the effects produced in a field of force. The characteristics of a field are known when we know the direction and intensity of the forces exerted at various points in it.

It must be remembered that no *force* exists in a field except when there is present in it some body or agent peculiar to that field. For example, in a gravitation field no force exists except where *matter* is present. In a magnetic field no force exists except where a *magnetic pole* is present. The *intensity* of a field at any point is measured by the force which would act upon a *test unit* of the kind to which the force is due, if such a unit were present. The actual force exerted is the product of the field intensity by the number of such units present.

Now in studying the effects of such forces where their directions and intensities vary from point to

point within the field, the problem becomes very complicated if we attempt to solve it by taking into account the forces themselves.

But every movement in a field of a body acted upon by the forces involves *work*, and problems relating to the effects of such movements are much simplified if we express the characteristics of the field in terms of the work done in moving a body from one point to another in it, instead of in terms of the direction and intensity of the forces.

Difference of Potential.—We define the *difference of potential* between two points in a field of force to be a difference of condition between the two points which is measured by the work which would be done by the forces in the field in moving a test unit from one point to the other. The work done in this case is independent of the *path* over which the body is moved. For it is self-evident that if work is done *by* the forces in moving a body from the point A to the point B in a field of force, exactly the same work must be done *against* the forces in moving the body back by the *same path* from B to A. Now if more work can be done by the forces of the field by moving the body over one path than by moving it over another, the body might be made to move *from* A to B by the path giving the greater work, and *back* to A by the path requiring the lesser work, and so work could be continually done by simply allowing a body

to go from one point to another by one path, and back to the first point by another. But this is inconsistent with the principle of the conservation of energy.

If V and V_1 represent the potentials at the points A and B respectively, and s the distance between them, it is plain that the average force that is exerted upon a test unit as it moves from one point to the other is $F = \frac{V - V_1}{s}$, and if the points are taken very near together F will be the force at the middle point between them. But $\frac{V - V_1}{s}$ is the *rate of change* of potential with respect to space. Hence the intensity of the field at any point is the *rate of fall of potential* at that point.

Electrical Difference of Potential.—An electrically charged body produces a field of force, as indicated by the action upon light bodies.

Two kinds of electrical charges. One charge can never exist without the other.

Bodies having like charges repel, having unlike charges attract.

Charges communicated by contact.

A light body vibrates between two bodies oppositely charged. It is evident that *work is done* by the electrical forces producing this vibration.

Measurement of Potential Difference.—The dif-

ference of electrical potential between two charged bodies is equal to the work done in carrying a small body charged with the *unit quantity* of electricity from one to the other, and the work done in carrying any charged body from one point to another is measured by the product of the quantity of electricity upon the body carried and the difference of potential between the two points.

If two charged bodies be connected by a metallic wire, they will, unless they are connected with some electric generator, be discharged or brought to equilibrium. Electricity is said to flow from one to the other. This constitutes an electric current. If the charges upon the bodies are maintained by connecting them with some electric generator, a continuous current flows through the wire. In this case, as in the case of the vibrating body, the work done is measured by the product of the quantity of electricity carried by the wire, and the potential difference between the two bodies, or

$$W = EQ, \quad . \quad . \quad . \quad . \quad . \quad (26)$$

where W is the work done, and E the potential difference.

The c.g.s. unit potential difference may now be defined as that potential difference which, in transferring the c.g.s. unit quantity of electricity, performs

work equal to one erg. The *practical* unit potential difference is the *volt* = 10^8 *c.g.s. units.*

The rate of working or *power* expended is

$$P = \frac{W}{t} = \frac{EQ}{t},$$

or, since $\frac{Q}{t} = C$,

$$P = EC. \qquad \ldots \ldots \quad (27)$$

Since the volt is 10^8 c.g.s. units and the ampere is 10^{-1} c.g.s. units, one ampere flowing with a fall of potential of one volt gives 10^7 ergs per second, or one watt.

The test of electrical difference of potential is the attraction or repulsion of light bodies, or, in the case of charged bodies, the current produced when the bodies are connected by a conductor.

Gold-leaf electrometer.

Quadrant electrometer.

Electromotive Force may be defined as anything that produces or tends to produce an electrical flow. Under this definition, potential difference is an electromotive force, but electromotive force, E.M.F., is a much broader term than difference of potential. It includes all agencies that tend to produce an electric flow. The term electromotive force is usually applied to such agencies as tend to disturb the electrical equilibrium and bring about difference of potential.

When a glass rod is rubbed with silk an electromotive force is developed which transfers electricity from the silk to the rod and develops a difference of potential.

Difference of potential is *always* due to a disturbance of electrical equilibrium by an electromotive force.

Electricity, like water, seeks its own level, and left to itself, differences of potential would sooner or later disappear.

E.M.F. is measured by the difference of potential it can produce.

Examples of E.M.F.—When copper and zinc plates are immersed in dilute sulphuric acid the copper becomes positive and the zinc negative. An E.M.F. exists, tending to carry electricity across the liquid from the zinc to the copper. This E.M.F. is independent of the size of the plates.

When the junction of two metals is heated, an E.M.F. is in general developed which carries electricity from one metal to the other, producing a difference of potential between them.

When a conductor is moved across a magnetic field, an E.M.F. in general exists, causing a transfer of electricity from one end toward the other, so developing a difference of potential between the two ends.

PROBLEMS.

(16) A potential difference of 200 volts exists between two bodies 10 cm. apart. What is the mean force acting upon a small body charged with one coulomb to carry it across from one body to the other?

(17) A 16-candle incandescent lamp consumes about .5 ampere at 110 volts. How many watts? How many horse-power to operate 500 such lamps?

(18) How much heat may be developed by a current of 6 amperes at 240 volts?

(19) What quantity of electricity is conveyed in one hour by a current of 10 amperes?

(20) A body charged with 50 coulombs is discharged in 10 seconds. What is the mean current?

RESISTANCE AND OHM'S LAW.

There is no such thing as a *perfect* conductor of electricity.

That the best conductors offer *resistance* to the flow of electricity in them is shown by the fact that whenever a current flows there is always a fall of potential along the conductor.

Resistance is proportional to the *length* of the conductor and inversely proportional to its *cross-section.*

Specific Resistance.—Different materials forming conductors of same length and cross-section vary greatly in resistance. The specific resistance of a substance may be defined as the ratios of the resistance of a conductor of that substance to the resistance of a conductor of same length and cross-section of some other substance, taken as a standard.

Or, the *absolute* specific resistance of a substance is the resistance of a centimetre cube of that substance taken between opposite faces.

Copper and silver have the least specific resistance. Other metals have varying specific resistances. Iron has about six times and mercury about sixty times the specific resistance of silver.

Liquids have much higher resistances. The resistance of the liquids used in galvanic batteries is from one to ten million times that of copper.

Ohm's Law states that the current flowing in a conductor is directly proportional to the potential difference, and inversely proportional to the resistance, or, since we have yet to choose a unit of resistance, we may put

$$C = \frac{E}{R} \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

Unit Resistance.—The c.g.s. unit resistance may now be defined as that resistance through which the c.g.s. difference of potential will carry the c.g.s. unit current.

The practical unit resistance is the *ohm* $= 10^9$ c.g.s. units.

Hence amperes $= \frac{\text{volts}}{\text{ohms}}$.

The megohm = a million ohms, and the microhm = one millionth ohm, are units often used.

Conductivity measures the capacity of a conductor to carry current. It is the reciprocal of *resistance*.

Insulation Resistance.—No substance is a *perfect* insulator. Gutta percha, one of the best insulators, has a resistance 85×10^{19} times the resistance of copper.

The insulation of telegraph lines and cables, and of

insulated wires generally, is given in megohms per mile.

Value of the Ohm.—One ohm is equal to the resistance of a column of mercury one millimetre in cross-section and 106.3 cm. long.

It is roughly the resistance of a copper wire $\frac{1}{25}$ inch in diameter and 50 yards in length.

Power Consumed by Resistance. — Combining equations (27) and (28), we have

$$P = C^2R; \quad . \quad . \quad . \quad . \quad . \quad (29)$$

$$P = \frac{E^2}{R}. \quad . \quad . \quad . \quad . \quad . \quad (30)$$

This means that in a given conductor the electrical power consumed is proportional to the square of the current, or to the square of the fall of potential along that conductor. Since the electrical energy expended in a conductor develops heat, the heating effect of a current in a given conductor is proportional to the square of the current.

Divided Circuits.—When the terminals of an electric generator or any two bodies between which a difference of potential is maintained are joined by two or more conductors, current flows through each of them in accordance with Ohm's law.

Conductors so connected are said to be joined in

multiple arc, or *in parallel*. Figs. 5 and 6 are typical representations of such an arrangement.

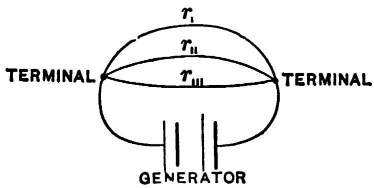

FIG. 5.

If E be the difference of potential between the two bodies to which the conductors are joined, and $r_{\prime}$, $r_{\prime\prime}$,

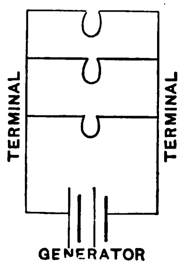

FIG. 6.

$r_{\prime\prime\prime}$, etc., are the resistances, respectively, of the several conductors, the currents flowing will be

$$C_{\prime} = \frac{E}{r_{\prime}};$$

$$C_{\prime\prime} = \frac{E}{r_{\prime\prime}};$$

$$C_{\prime\prime\prime} = \frac{E}{r_{\prime\prime\prime}};$$

etc. etc.

Evidently the total current is

$$C=\frac{E}{r_{,}}+\frac{E}{r_{,,}}+\frac{E}{r_{,,,}}+\text{etc.}=\frac{E}{R}, \quad . \quad . \quad (31)$$

where R is the *equivalent* resistance of the several conductors. From (31) R can be calculated when $r_{,}$, $r_{,,}$, etc., are known. For example, (31) gives for three conductors

$$\frac{1}{r_{,}}+\frac{1}{r_{,,}}+\frac{1}{r_{,,,}}=\frac{1}{R},$$

whence

$$(r_{,}r_{,,}+r_{,}r_{,,,}+r_{,,}r_{,,,})R=r_{,}r_{,,}r_{,,,};$$

$$R=\frac{r_{,}r_{,,}r_{,,,}}{r_{,}r_{,,}+r_{,}r_{,,,}+r_{,,}r_{,,,}}. \quad . \quad . \quad (32)$$

When two conductors are connected in multiple, the fall of potential is the same along both. Evidently for every point on one there must be a point on the other having the same potential. If two such points be connected by a wire, no current will flow through that wire.

In Fig. 7 let *ABD*, *ACD*, be two conductors joining the terminals of the generator *S*. The point *C* on *ACD*, which has the same potential as *B* on *ABD*, may be found by connecting *B* to one terminal of a delicate current-indicator *G*, to the other terminal of which a wire is connected which may be slid along *ACD*. When *G* indicates no current, the point *C* is found.

Let r, $r_{,}$, $r_{,,}$, $r_{,,,}$, be the resistances of the several sections of the conductors as marked on the figure.

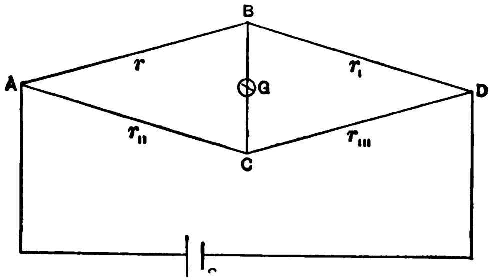

FIG. 7.

Let e be the fall of potential from A to B and from A to C. Let $e_{,}$ be the fall of potential from B to D and from C to D. Then, since the same current flows through AB and BD,

$$\frac{e}{r} = \frac{e_{,}}{r_{,}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

For a similar reason

$$\frac{e}{r_{,,}} = \frac{e_{,}}{r_{,,,}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

Dividing (34) by (33),

$$\frac{r}{r_{,,}} = \frac{r_{,}}{r_{,,,}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

which shows the relation between the four resistances.

PROBLEMS.

(21) Two conductors in multiple arc have resistances of 24 and 30 ohms respectively. What is the

equivalent resistance? A current of 8 amperes flows in the circuit. What is the potential difference between the ends of the conductors? What current flows through each?

(22) Four conductors, of 1, 2, 3, 4 ohms respectively, are in multiple. What is the equivalent resistance?

(23) A galvanometer has a resistance of 4500 ohms; it is desired to place in parallel with it a resistance that shall shunt away from it $\frac{9}{10}$ the current. What must be that resistance?

(24) What is the equivalent resistance of 500 incandescent lamps in parallel, each lamp having a resistance of 200 ohms?

(25) What current will the lamps of the last problem consume at a potential difference of 110 volts? Suppose the leads from the generator to the lamps have a resistance of 0.01 ohm. What must be the potential difference at the generator to maintain 110 volts at the lamps?

(26) If the incandescent lamps of problem (24) are connected five in series and these groups connected in multiple, what is the equivalent resistance? What current will be consumed if each lamp carries the same as before? What potential difference between the supply leads will be necessary?

PRACTICAL MEASUREMENTS OF ELECTRICAL QUANTITIES.

In the preceding lectures the relations between the several electrical quantities have been brought out, and the bases upon which the several units of measurement have been defined and established have been fully discussed. It is, of course, possible to measure these electrical quantities by methods based upon those relations and definitions. Measurements by such methods are called absolute measurements.

When treating of electric currents it was shown how the value of a current could be determined in absolute measure by means of the tangent galvanometer. The methods for the absolute measurement of potential and resistance are, however, too tedious and complicated to be made use of for general measurements, and are only resorted to for the purpose of constructing standards with which the quantities to be measured are thereafter compared. In the measurement of currents, even, the absolute methods are unsuitable for general work, and standards have been devised which render unnecessary the absolute determination of any current. After comparing the results of the

most accurate absolute determinations the Electrical Congress held in Chicago in 1893 fixed upon the following as the physical representatives of the electrical units:

"As the Unit of Current, the International Ampere, which is one tenth of the unit current of the c.g.s. system of electromagnetic units, and which is represented sufficiently well for practical use by the unvarying current which when passed through a solution of nitrate of silver in water and in accordance with the accompanying specification deposits silver at the rate of 0.001118 grammes per second."

Similarly the International Volt is declared to be represented sufficiently well "by $\frac{1000}{1434}$ of the E.M.F. between the poles or electrodes of the voltaic cell known as Clark's cell at a temperature of 15° C., and prepared in the manner described in the accompanying specification."

And the International Ohm is declared to be "represented by the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grammes in mass, of a constant cross-sectional area, and of the length of 106.3 centimetres." The cross-sectional area of such a mass of mercury 106.3 cm. in length, is one square millimetre.

But it is not convenient in ordinary measurements to make use of these official representatives of the

units. For practical use instruments are constructed by which the electrical quantities may be measured much as we measure length by the foot-rule or tape-measure, or mass by the balance and weights. Ordinarily we accept the accuracy of such instruments as we accept the accuracy of the weights of the balance, upon the reputation of the manufacturer; but it must be remembered that the electrical instruments, especially those for measurement of potential and current, are likely to change with time, and all are liable to accidental derangements, and it is not safe to trust to their accuracy as we trust to the accuracy of the foot-rule or tape-measure for an indefinite period. For all important measurements the instruments employed should be carefully tested by comparison with standards of known accuracy.

MEASUREMENTS OF RESISTANCE.

Instruments.—Certified standard resistances.

Resistance sets. These are sets of resistance coils so arranged that any resistance from that of the smallest coil to the sum of all the resistances in the instrument may be employed at pleasure. There are two principal arrangements: coils of 1, 2, 2, 5, 10, 20, 20, 50, etc., ohms, or ten unit coils, ten 10-ohm coils, etc., are arranged in series, with provision for cutting in or out of circuit any desired portion.

Methods of Measurement.—*First. By direct comparison with known resistances:*

(*a*) By *substitution.* This consists in noting the deflection of a galvanometer when connected in circuit with the unknown resistance, then substituting for the unknown resistance known adjustable resistances, and adjusting these until the same deflection is obtained. The known resistance is then equal to the unknown.

(*b*) By the *differential galvanometer.* The differential galvanometer is an instrument having two coils of equal resistance, so adjusted as to have exactly the same influence upon the needle. With equal currents flowing in opposite directions in these two coils the needle would be undisturbed. To use the instrument, the unknown resistance is connected in circuit with one coil, and adjustable known resistances in circuit with the other, the two circuits being connected in multiple arc between the terminals of some electric source. When the known resistances are so adjusted that the galvanometer needle suffers no deflection, the known and unknown resistances are equal.

Second. By fall of potential. This is a method especially applicable to the measurement of small resistances. The resistance to be measured is connected in circuit with a galvanometer which will measure the current flowing. The difference of po-

tential between the two ends of the resistance is then measured by means of some instrument for measuring potential differences. If C be the current and e the difference of potential, then, from Ohm's law,

$$R = \frac{e}{C}.$$

The figure below illustrates the arrangement. R is the resistance to be measured, G the galvanometer, and V the potential instrument.

Third. By Wheatstone's bridge. This is an apparatus utilizing the principle illustrated in Fig. 7.

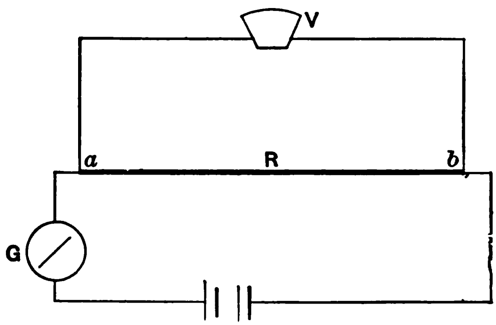

FIG. 8.

Suppose any one of the resistances of Fig. 7 to be unknown; it can be determined from equation (35). As the instrument is usually constructed, two of the resistances, as $r_{,}$, $r_{,,}$, have a simple ratio, as 1 : 1, 1 : 10, 1 : 100. A third resistance, r, is known and adjustable. The unknown resistance is then con-

nected as $r_{,,,}$. The instrument is often called Wheatstone's *balance;* $r_{,}$ and $r_{,,}$ are called the *arms* of the balance. The manipulation consists in varying the resistance r until the needle G is undisturbed by the closing of the circuit.

Measurement of Insulation Resistance.—Insulation resistances up to ten megohms may be measured by means of the Wheatstone's bridge, but for the measurement of very high insulation resistances it is customary to use a delicate galvanometer which will give a deflection equal to one scale division for a difference of potential of one volt through a determined resistance of several megohms. This galvanometer is merely put in circuit with the insulation resistance to be measured, and the deflection for a given potential difference noted.

Measurement of Very Small Resistances.—For such measurements methods must be employed by which the resistances of the connections and contacts by which the resistance to be measured is connected to the apparatus are eliminated. The fall of potential method illustrated in Fig. 8 permits this. The connections a, b to the potential instrument V are so made as not to include the contact resistances by which the battery is connected to the resistance R.

The method as before described necessitates the accurate observation of current and potential. The

arrangement shown in Fig. 9 does away with the necessity of observing either of these quantities, but permits, instead, the direct comparison of the resistance to be measured with a known adjustable resistance. *A* is the unknown resistance, *R* a known

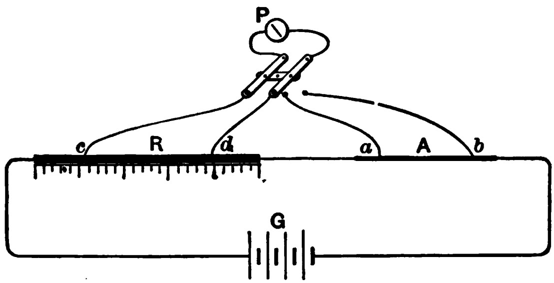

FIG. 9.

resistance, such as a graduated wire. *P* is a delicate galvanometer, the value of whose indications need not be known. The operation consists in sliding the contacts *cd* along *R* until *P* shows the same deflection whether connected with *R* or *A*. The resistance between *a* and *b* is then the resistance between *c* and *d*.

Measurement of Resistance of Electrolytes.—When an electric current flows through an electrolyte it not only does work in overcoming the true resistance, but it also does work in decomposing the electrolyte. This latter work is done in overcoming what is called the counter-electromotive force of the liquid. This is an apparent resistance which is independent

of the dimensions of the liquid column, and depends only upon the nature of the liquid, assuming the liquid to have no action upon the electrodes. Means by which this apparent resistance may be eliminated must be employed for measuring the true resistance.

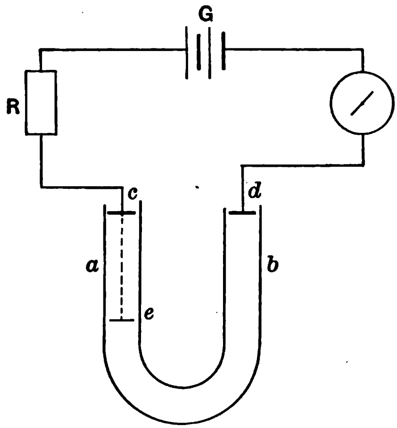

FIG. 10.

Fig. 10 shows one method. *ab* is a U tube containing the electrolyte, *G* is a battery, and *R* an adjustable resistance; *c* and *d* are platinum plates nearly filling the tube. Let *R* be adjusted until the galvanometer gives a convenient deflection. Now let one of the platinum plates be lowered a measured distance—to *e*, say. Now adjust *R* until the galvanometer shows the same deflection as before. The increase in *R* is the resistance of the column of liquid *ce*.

If a column of an electrolyte is traversed by a rapidly alternating current, no permanent decomposition occurs, and no work is done except in overcoming the true resistance. The resistance may then be measured by any of the methods before described for measuring the resistance of ordinary conductors, but, instead of the galvanometer, an electrodynamometer or some instrument affected by alternating currents must be used. If a Wheatstone's bridge is employed with alternating currents for measuring the resistance of an electrolyte, a telephone may conveniently be used in place of the galvanometer to indicate when a balance is obtained.

The resistance of an electrolyte varies greatly with temperature and, if a solution, with the degree of concentration. Measurements are of no value unless these conditions are noted.

Resistance of Batteries.—The measurement of the resistance of battery-cells presents some difficulties on account of the electromotive force. The following are some of the methods employed:

(*a*) Two similar cells are connected by two like poles so that their electromotive forces are opposed. Their joint resistance may then be measured as in the case of ordinary conductors.

(*b*) The potential difference between the poles of the cell on open circuit is measured. This gives the E.M.F. of the cell. Represent it by E. The circuit

of the cell is then closed through a known resistance r, and the potential difference between the poles again measured. Call this e. If R be the resistance of the cell, the current flowing is

$$C = \frac{E}{R + r} = \frac{e}{r}.$$

$$rE = eR + er;$$

$$R = \frac{E - e}{e} r. \quad . \quad . \quad . \quad . \quad . \quad (36)$$

If $e = \frac{1}{2}E$, $\qquad R = r.$

Hence we may observe the potential of the cell on open circuit, then close the circuit through a resistance which is so adjusted as to reduce the potential to one half. The known resistance in circuit is then equal to the resistance of the cell.

This method assumes that the E.M.F. of the cell remains constant during the measurements. This is not necessarily true.

(*c*) By Mance's method, in which the cell whose resistance is to be measured is connected as the unknown resistance in the Wheatstone's bridge, as in the diagram, Fig. 11. The key K is inserted in place of the usual battery. The galvanometer G must be one whose deflection will not be too great. It may be necessary to put a known resistance in the branch X in series with the cell, to obtain a convenient deflection. Having chosen a convenient ratio for the arms

a and b, R is adjusted until the deflection of the galvanometer G is unchanged by opening and closing the key K. It can be shown that, when this condi-

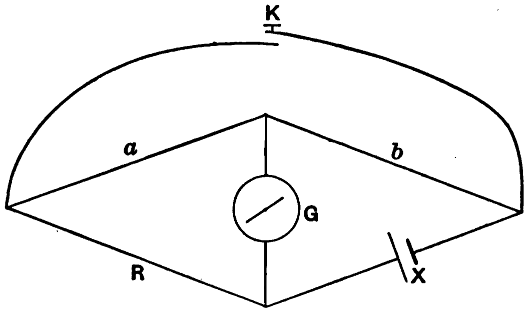

FIG. 11.

tion is satisfied, the usual relation between the resistances of the bridge branches exists. That is,

$$\frac{a}{b} = \frac{R}{x}.$$

From this X is determined. If a known resistance was connected in series with the battery-cell in order to obtain a suitable deflection of the galvanometer, this must be subtracted from X to give the cell resistance.

MEASUREMENT OF CURRENT.

Measurements of current by means of the tangent galvanometer have already been fully explained. Other current-measuring instruments have been briefly

described. These are usually adjusted by the manufacturers to read directly in amperes, and it is only necessary to connect the instrument in the circuit where the current is to be measured, and note the reading. It is important in the use of all such instruments to place them where they will be uninfluenced by outside magnetic forces.

Current Measured by Fall of Potential.—In a conductor carrying a current there is always a fall of potential between its ends. This fall for the same conductor is proportional to the current, hence may be taken as a measure of the current. Fig. 12 shows

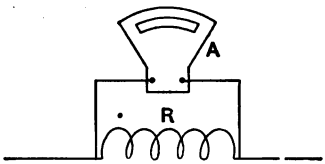

FIG. 12.

diagrammatically the arrangement of an instrument designed to utilize this method of measuring currents. *R* is the fixed resistance and *A* an instrument whose indications are proportional to the potential differences between its terminals. It is not necessary to know the resistance *R*. The instrument may be calibrated by direct comparison with some standard current-measuring instrument.

Examples of such instruments.

MEASUREMENT OF POTENTIAL.

By Electrostatic Forces.—This consists essentially in the measurement of the electrostatic forces existing between two bodies charged to the difference of potential to be measured. Instruments designed for such measurements may be so constructed that the potential difference may be computed from their dimensions and the forces observed. This is an absolute measurement, but is applicable only to the measurement of large potential differences.

The Quadrant Electrometer.

The Attracted-disk Electrometer.

By Comparison with a Known Potential Difference.—Fig. 13 illustrates this method. *ab* is a

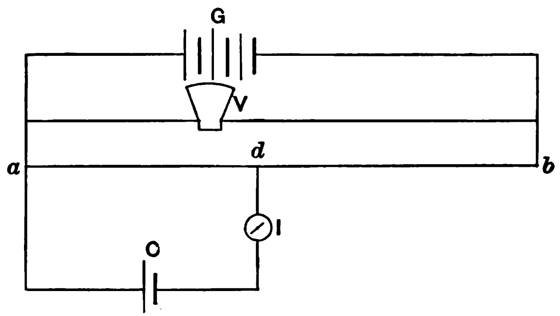

FIG. 13.

graduated resistance, such as a long wire or a series of resistance coils. *G* is a generator capable of main-

taining a constant potential difference between a and b, which must be known. C is a battery-cell whose E.M.F. is to be measured, so connected in the branch aCd that its E.M.F. is opposed to the E.M.F. acting along that branch in consequence of the fall of potential along ab. Now the point of contact d is moved along the resistance ab until the galvanometer I is undeflected. The potential difference between a and d now balances the E.M.F. of the cell C. If E be the potential difference between a and b, and E_1 the E.M.F. of the cell C, we have

$$\frac{\text{res. } ab}{\text{res. } ad} = \frac{E}{E_1}.$$

An instrument so constructed as to provide for the convenient application of the above method is called a *potentiometer*.

Different forms of potentiometer.

By the Current produced in Circuit of Given Resistance.—It follows from Ohm's law that E.M.F. is proportional to the current it develops in a given resistance. Hence the readings of any galvanometer are proportional to the potential difference between its terminals. If a galvanometer of high resistance be connected between two points differing in potential, its indications will be proportional to that potential difference, and the instrument may be graduated

to give the potential difference directly in volts. It is then called a *voltmeter*.

Different forms of voltmeter shown and described.

Since closing a circuit between two points may alter the potential difference between those points, the use of such voltmeters as are described above is inadmissible, except where the generating source is such as to maintain that potential difference notwithstanding the current consumed by the voltmeter.

By the Ballistic Galvanometer.—If two bodies, as the two coatings of a Leyden jar or other condenser, be connected to two points of an electric circuit, those bodies will be charged to whatever potential difference may exist between those points. If they are afterward connected through a suitable galvanometer, they will be discharged, and the discharge-current flowing through the galvanometer coil will give the needle a sudden impulse, causing it to swing off a certain distance depending upon the quantity of electricity discharged, and this again is proportional to the potential difference and to the *capacity* of the condenser receiving the charge.

In this way potential differences may be measured by means of the galvanometer without actually forming a circuit and taking current from the source.

A galvanometer suitable for this purpose is one whose needle swings freely, with the smallest possible retarding influence, so that its needle once disturbed

will swing for a long time before coming to rest. It should also have a slow rate of vibration. Such an instrument is called a *ballistic galvanometer*.

TESTING AND CALIBRATING INSTRUMENTS.

Electrical instruments are liable to derangement and must be frequently tested to determine whether changes have occurred. This is especially true of instruments for the measurement of current and potential. Where a high degree of accuracy is required, instruments must be compared with accredited standards, their errors determined, and the necessary corrections applied when the instruments are used for measurements.

Tests of Resistance Sets.—It is seldom required to reproduce the mercury standard ohm for purposes of comparison. A certified standard one-ohm coil may be used instead. The one-ohm coil of the set is compared with the standard. Then this one-ohm coil plus the standard is compared with the two-ohm coil of the set, and so on until all are compared. In a decade set each one-ohm coil would be compared with the standard, then the ten one-ohm coils in series with each ten-ohm coil, etc.

Method of Comparison.—A form of Wheatstone's bridge known as the divided-metre bridge is usually employed for comparing resistances. Fig. 14 shows the arrangement. AB is a wire of as nearly as possi-

ble uniform cross-section. G, G_1, etc., are heavy copper strips. E and F are two nearly equal resistances, which, to give the greatest sensibility, should be about the value of the resistances to be compared, which are connected in the two gaps C, D. I is now moved along the wire AB until the galvanometer is undeflected by closing the circuit. The resistances at C and D are now interchanged. It is plain that, if they are equal, the galvanometer will still be un-

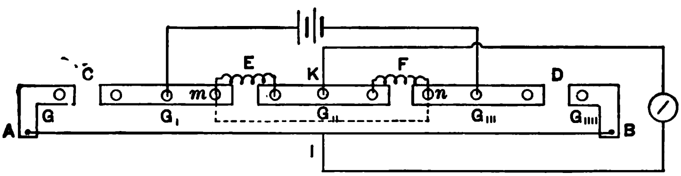

FIG. 14.

deflected by closing the circuit. If C and D are not equal, I will have to be moved in order to obtain a balance after the interchange. It is easy to show that the resistance of the bridge-wire between the two positions of I equals the difference between the resistances of the two coils that are being compared. The bridge-wire AB in the standard form of this apparatus is one metre in length, and under it is a scale divided into millimetres; hence the name. If the resistance of one millimetre of the bridge-wire is known, the difference in fractions of an ohm between two coils under comparison is at once given.

Calibration of the Bridge-wire.—It is rarely the case that the resistance of the bridge-wire is uniform. To determine its resistance in different parts of its length it must be calibrated. To effect the calibration the resistances E and F are removed, and the points m and n are connected by a wire as shown by the dotted line. The galvanometer connection K is now made on the wire mn by a movable contact-piece. Place at C a standard one-ohm coil, and at D a 1.01-ohm standard; then effect a balance by moving either I or K. Now interchange C and D and balance again by moving I. Evidently the resistance of the wire between the two positions of I is .01 ohm. Return C and D to their former positions, and, without moving I, balance by sliding K along mn. Again interchange C and D and balance by moving I. This will give another .01-ohm section of AB. Continuing this process step by step, the whole wire may be subdivided into .01-ohm sections, from which the resistance between any two points may be obtained.

In making such tests as those described, arrangements must be made for maintaining a constant temperature, and the connections at C and D must be so made that their resistances are extremely small and are invariable during all the interchanges.

Tests of Current Instruments.—*First.* Comparison with a standard. The instrument is connected in circuit with a silver voltameter and the test conducted

as directed on page 106 of "Physical Units" by Professor Magnus Maclean. This gives the true value of the reading for one point of the scale.

Second. Comparison of different scale-readings. Having found the value of the scale-reading for one point, the instrument is connected in circuit with a variable resistance whose values can be accurately determined, and a source of electricity giving a constant difference of potential. The constancy of the potential may be determined by the use of any potential indicator. It is not necessary that its value should be known. The test consists in varying the current by means of the variable resistance, and noting the resistance and corresponding readings of the instrument.

Tests of Instruments for Measuring Potential.—Voltmeters may be tested by means of the potentiometer and the standard Clark cell. Referring to Fig. 12, *ab* is the potentiometer resistance, which should be as much as 10,000 ohms. *V* is the voltmeter to be tested, and *C* the standard cell. The generator *G* should have an E.M.F. greater than the highest reading of the voltmeter, and it will be convenient if its E.M.F. is adjustable. If this is not the case, the potential difference between *a* and *b* may be varied by shunting a part of the current away from *ab*.

The test consists in varying the potential difference

between *a* and *b* until the voltmeter gives a convenient reading. Then move the connection *d* until *I* is undeflected. We then have

$$\left.\begin{array}{l}\text{value of volt-}\\ \text{meter reading}\end{array}\right\} = \frac{\text{res. } ab}{\text{res. } ad} \times \text{E.M.F. of Clark cell.}$$

The E.M.F. of a Clark cell for any temperature t is

$$1.434[1 - 0.00077(t - 15)].$$

Clark cells are themselves liable to derangement. It is best to have several of them, and compare them with each other by means of the potentiometer. If two or three of them agree at a given temperature, it is safe to assume that their E.M.F. at 15° C. is 1.434 volts.

When using a Clark cell it is necessary to place in series with it a protecting resistance of several thousand ohms to prevent passing through it too large a current. This resistance may be shunted for the purpose of obtaining a higher sensitiveness when the potentials are nearly balanced.

HEATING EFFECTS OF THE CURRENT.

When a current flows through a homogeneous conductor, that conductor is always heated.

Equation (29) shows that the energy spent in such a conductor per second is C^2R—directly proportional to the resistance, and to the square of the current. C being given in amperes and R in ohms, the heat generated is $.00024C^2R$ calories per second $= .000948C^2R$ British thermal units per second.

A 16-candle incandescent lamp consumes about 0.5 ampere at 110 volts. Its resistance is, therefore, 220 ohms. The energy expended in it is $110 \times .5 = 220 \times .5^2 = 55$ watts $= 55 \times .00024 = .0132$ calorie per second $= .074$ H. P.

Temperature of a Conductor carrying Current.—The temperature to which a conductor is raised by the current depends not only upon the rate of development of heat in it, but upon the rate at which heat escapes. The temperature becomes permanent when heat escapes as fast as it is generated.

For a round wire of given length the resistance is proportional to the inverse square of the diameter, while the radiating surface is directly proportional to

the first power of the diameter. The heat generated by the *same current* in a wire whose diameter is d, as compared to that generated in a wire whose diameter is unity, is, therefore, $\frac{1}{d^2}$, while the surface from which heat escapes is $\frac{d}{1}$ times as great. The heat which must escape from unit surface of the larger wire is, therefore, $\frac{1}{d^3}$ of that which must escape from unit surface of the smaller wire. If heat escaping per unit surface is proportional to the difference of temperature between the wire and its surroundings, the larger wire will rise in temperature only $\frac{1}{d^3}$ as much as the smaller.

Equation (30) shows that the heat developed in a wire is proportional to $\frac{E^2}{R}$.

This means that for the *same difference of potential* the heat generated in conductors of different resistances is inversely proportional to the resistance, and, since resistance for wires of the same length is proportional to the inverse square of the diameter, the heat generated in such wires when subjected to the *same potential difference* is proportional to the square of the diameter directly. Hence if two wires, diameter unity and diameter d, are connected in multiple between the terminals of an electric generator, d

times as much heat will be generated in the wire whose diameter is *d*. But its surface is only *d* times as great, hence *d* times as much heat must escape from unit surface, which requires that it rise in temperature *d* times as much. It follows that a coarse wire will be heated to a higher temperature than a fine one of the same length when connected across a circuit where the difference of potential is the same.

INCANDESCENT LIGHTING.

Description of the incandescent lamp.

Incandescent lamps are usually connected in multiple arc across a circuit from a generator capable of maintaining a constant difference of potential, whether lamps in use are few or many.

Fig. 6, on page 39, may be taken as a diagrammatic illustration of such a system.

Economy requires High Temperature.—Economy in incandescent lighting requires that the carbon filament shall be maintained at as high a temperature as it will bear without a too-rapid deterioration, for the reason that the ratio of light emitted to energy consumed increases very rapidly as the temperature increases. At the best, only about 5 per cent of the energy expended in the lamp appears as light; the remaining 95 per cent is dark heat, useless for illumination, and therefore wasted.

Incandescent Lamps of Different Candle-power.—Since potential is constant, the power expended in an incandescent lamp is inversely proportional to its resistance (see equation (30)).

Let it be required to construct a 32-candle lamp, that is, a lamp of twice the usual illuminating power. This requires that twice the power be expended; hence that the resistance be reduced to one half. Since the temperature must remain constant, the radiating surface must be doubled.

Let l be the length, b the width, and d the thickness of the 16-c.p. filament, and let $l_1 b_1 d_1$ be the same for the 32-c.p. filament. Resistance is proportional to $\frac{l}{bd}$; hence

$$\frac{l}{bd} = 2\frac{l_1}{b_1 d_1}. \quad . \quad . \quad . \quad . \quad . \quad (a)$$

Radiating surface is proportional to $l(b + d)$; hence

$$2l(b + d) = l_1(b_1 + d_1). \quad . \quad . \quad . \quad . \quad (b)$$

In these two equations there are three unknown quantities. The values are therefore indeterminate; but if we assume the value of one, the other two are fixed. Assume, for example, $l_1 = l$; then from (a)

$$b_1 d_1 = 2bd;$$

from (b)

$$b_1 + d_1 = 2(b + d).$$

That is, we must double the cross-section and also double the periphery. To do this requires that the ratio of width to thickness must be changed, as will be seen by finding the values of b_1 and d_1 from the two equations above.

ARC LIGHTING.

Description of arc lamp.

Light mainly proceeds from a small intensely heated surface of the positive carbon, called the crater, from which the arc springs. Some comes from the red-hot ends of carbons. Very little comes from the arc itself.

Source of the Light.—In almost all arc lamps used for illumination the upper carbon is made positive and the crater is a small concave surface at its end. Light can only issue from this in a direction obliquely downward. Hence an arc lamp gives a very unequally distributed illumination. A so-called 2000-candle arc lamp gives its most intense light—about 1500 to 2000 candles—at an angle of 45° to 60° below the horizontal. From this direction the intensity rapidly diminishes until, on the horizontal plane through the lamp, the illumination amounts to only 200 to 300 candles.

As a result, a lamp suspended at a height of, say, 25 feet gives a most intense illumination on the ground

below over a circle of about 50 feet in diameter, beyond which the illumination rapidly diminishes.

The so-called 2000-candle arc lamp consumes about 10 amperes at 45 volts, or 450 watts. Of this 45 volts, about 39 volts seems to be expended on the surface of the crater, the remainder being expended in overcoming the resistance of the arc. That is, about seven eighths of the power is expended in the crater.

About 10 per cent of the power expended appears as light, the remaining 90 per cent appearing as dark heat.

The light produced by an arc lamp depends almost wholly upon the current, very little upon the potential difference so long as this is above 40 volts. Upon the latter depends the length of arc or the amount of separation of the carbons. About 45 volts is required to maintain a separation that will permit the free emergence of the light from the crater. Beyond this very little is gained by increase of potential.

Arc lamps are usually connected in circuit *in series;* hence the same current flows through all of them. The E.M.F. required for the operation of such a circuit is the E.M.F. required for one lamp multiplied by the number of lamps.

Enclosed arc lamps.

PROBLEMS.

(27) If a round filament 4 inches long, .012 inch diameter, is suitable for a 16-c.p. lamp, what are the dimensions of a similar filament suitable for 32 c.p.?

(28) Fifty arc lamps, requiring a current of 10 amperes at 45 volts each, are connected in series in a circuit 5 miles long, the conducting wire having a resistance of 0.25 ohm per thousand feet. What must be the E.M.F. of the generator?

(29) What energy (watts, horse-power) is consumed in the lamps of the last problem? What in the circuit?

(30) A 10-ampere, 45-volt arc lamp is connected in parallel with incandescent lamps across a 110-volt circuit. What resistance must be used in series with the lamp to reduce the potential to 45 volts? How much energy is expended? How much in the lamp?

(31) If two lamps like that of the last problem are placed in series across a 110-volt circuit, what resistance is required in series with them? How much energy is consumed? How much is wasted?

CHEMICAL EFFECTS.

Decomposition of Salts.—When two platinum plates immersed in dilute sulphuric acid are connected to the terminals of an electric generator, oxygen gas escapes in bubbles from the positive plate, and hydrogen from the negative plate. The relative proportions of oxygen and hydrogen are exactly the proportions in which they unite to form water.

In this experiment water is said to be decomposed by the current.

If the same plates be immersed in a solution of copper sulphate, copper is deposited on the negative plate, while oxygen escapes from the positive plate.

The operation here evidently is that the current separates the $CuSO_4$ into Cu and SO_4, the Cu appearing at the negative and the SO_4 at the positive plate. But SO_4 attacks and decomposes the water, liberating oxygen and forming sulphuric acid:

$$SO_4 + H_2O = H_2SO_4 + O.$$

If a copper plate be substituted for the positive platinum plate, oxygen no longer escapes, but the

SO_4 combines with the copper, forming $CuSO_4$, so maintaining the strength of the solution.

If the platinum plates are immersed in solution of sodium sulphate, Na_2SO_4, hydrogen escapes from the negative and oxygen from the positive plate, while the current passes, but at the same time caustic soda is found at the negative and sulphuric acid at the positive plate. The operation here is, Na_2SO_4 is separated into Na_2 at the negative and SO_4 at the positive plate, but Na_2 decomposes water,—

$$2Na + 2H_2O = 2NaHO + 2H,$$

the 2H escaping from the negative plate. At the same time SO_4 at the positive plate decomposes water, liberating oxygen there:

$$SO_4 + H_2O = H_2SO_4 + O.$$

Definitions of Terms.—The two plates forming the generator terminals are called *electrodes.* The positive plate by which the current *enters* the solution is called the *anode.* The negative plate by which the current *leaves* the solution is called the *cathode.* The solution which is decomposed is called the *electrolyte,* and the operation is called *electrolysis.* The two parts into which the electrolyte is separated by the current are called *ions.* Thus Cu and SO_4 are the two ions when solution of $CuSO_4$ is the electrolyte.

Ions appear only at Electrodes.—The ions appear only at their respective electrodes. No indication of any separation is found elsewhere, if the electrolyte is continuous from electrode to electrode.

Theory of electrolysis.

Faraday's Laws.—Faraday demonstrated, first, that the quantity of an electrolyte decomposed in a given time is proportional to the current *as measured by the tangent galvanometer;* second, that the quantity of any ion set free per second by a given current is proportional to its *chemical combining number*. Thus, one ampere sets free in one second

0.001118	grammes	of	silver,
0.0003279	"	"	copper,
0.0003367	"	"	zinc,
0.00001038	"	"	hydrogen,
0.11746 cc. of hydrogen.			

The combining numbers are:

Silver	107.7
Copper	31.59
Zinc	32.44
Hydrogen	1.

Because of this exact relation, which has been determined and verified by most careful experiments, the deposition of silver is taken as a means of standardizing current-measuring instruments.

Applications.—Electrotype.

Electroplating: Gold, silver, nickel.

Purification of metals by electro-deposition.

Energy required for Electrolysis.—The decomposition of a chemical compound is in general work that can only be accomplished by the expenditure of energy. Now work done by an electric current has been shown to be $W = EC$, where E is the E.M.F. or potential difference required to force the current C through the conductor or apparatus where the work is performed. For many chemical compounds the energy required for decomposition is well known, and since the current required is also known, the E.M.F. can be determined. For example, the combustion of one gramme of hydrogen develops 34 calories = 141440 joules. To decompose sufficient water to produce one gramme of hydrogen will require the same energy. To produce one gramme of hydrogen per second would require $\frac{1}{.00001038} = 96{,}340$ amperes nearly. Hence the E.M.F. required is $\frac{141440}{96340}$ = 1.47 volts, nearly. This is the *counter*-electromotive force of the electrolyte, and the decomposition of water with the visible formation of gas requires that this E.M.F., and as much more as is necessary to force the current through the liquid against its resistance, be supplied. The counter-E.M.F. is in-

dependent of the strength of the current, while the E.M.F. required to overcome resistance is proportional to the current and equal to CR. For the economical decomposition of an electrolyte CR should be small in comparison to the counter-E.M.F. That is, R must be made as small as possible.

When, as in most cases of electroplating, the anode is the same as the metal deposited, so that the composition of the electrolyte remains unchanged, the energy developed by the union of the negative ion with the anode exactly counterbalances the energy required to decompose the electrolyte. Hence the energy required for the operation in such cases is only C^2R, which goes to warm the electrolyte.

PROBLEMS.

(32) An apparatus for generating hydrogen and oxygen by means of the electric current, consisting of twelve decomposing cells arranged in series, furnishes 12 cubic feet of oxygen per hour. What current is employed?

(33) Silver, copper, and zinc voltameters are arranged in series on one circuit. A current of 20 amperes flows through them. How much metal per hour is deposited in each?

(34) To determine the constant of an ammeter, a current is passed through it and through a silver voltameter. A current giving a reading 25 deposits

104.6 grms. of silver per hour. What is the galvanometer constant?

(35) In an apparatus for electrical separation of silver, 30,000 (Troy) ounces of silver are deposited every 24 hours. What current is employed? What horse-power, assuming the E.M.F. to be 2 volts?

ELECTROMAGNETIC INDUCTION.

It has been shown that the movement of a wire across the lines of force in a magnetic field produces an E.M.F. If this wire form part of a closed circuit, a *current* will be produced, and this consumes energy.

Direction of Current.—The direction of the current produced is such that the movement of the conductor is *opposed* by the mutual action between the current and the field.

Examples of electromagnetic induction.

Currents result in all cases from a change in the number of lines of force threading through the circuit.

Law of Lenz.—When a current is induced by *any change whatever* in the relations between a conductor and a magnetic field, that current is in such sense as to oppose the change that produces it.

It follows that mechanical energy is consumed whenever a current is induced through the relative motion of a conductor and a magnetic field. From the law of conservation this energy must be the equivalent of the electrical energy developed.

Electromotive Force Developed by Electromagnetic Induction.—Suppose a current C and an

E.M.F. E are induced by the movement of a wire in a magnetic field. The electrical energy developed in time t is ECt. To move the wire must, by the law of Lenz, require a force F, and if in time t the wire moves parallel to itself through a distance s, the work done is Fs. From the last paragraph

$$Fs = ECt. \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

It has been shown that the force F exerted by a current C upon a magnetic pole is (see equation (20))

$$F = \frac{CLp}{d^2}.$$

This is also the force which the pole exerts upon the current, but $\frac{p}{d^2}$ is the intensity of field at distance d from a pole whose strength is p. Calling this H, we have

$$F = CLH. \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

This is the force exerted upon current C when placed in a magnetic field whose intensity is H. Substituting this value of F in (37),

$$CLHs = ECt,$$

or

$$E = \frac{HLS}{t}; \quad . \quad . \quad . \quad . \quad . \quad (39)$$

But L is the length of the moving conductor and s the distance it moves parallel to itself. Ls is then the

area swept through by it, and HLs the total number of lines of force cut by it. $\frac{HLs}{t}$ is then the number of lines of force cut per second. We have then the E.M.F. developed by a conductor moving in a magnetic field is numerically equal to the rate of cutting of lines of magnetic force by that conductor.

In order that the conductor of the above discussion may generate a current it must form part of a closed circuit. It is evident that the lines cut by the conductor are either added to or subtracted from the circuit of which the conductor forms a part. Hence, finally, the E.M.F. developed in a circuit by electromagnetic induction is numerically equal to the rate at which lines of magnetic force are added to or subtracted from that circuit. If the lines of force here considered are c.g.s. lines, the E.M.F. is in c.g.s. units. To reduce to volts, divide by 10^8.

ELECTROMAGNETISM.

It has been shown that a conductor carrying a current develops a magnetic field which may be represented by lines forming closed curves around the conductor.

In order that we may have a continuous current the conductor must be connected to the two terminals of a generator, and this, with the conductor, forms a closed electric circuit.

The electric circuit and the lines of magnetic force due to it therefore form two interlinking closed curves.

The Magnetic Circuit.—The intensity of the magnetic field set up by a current depends not only upon the strength of the current, but also upon the material that fills the space around the current and in which the magnetic lines must be formed. This material in which the magnetic lines are developed is considered as forming a *magnetic circuit*, akin to an electric circuit, and offering a greater or less opposition, depending upon the nature and dimensions of the materials composing it, to the development of

magnetic lines, just as an electric circuit offers more or less resistance to the flow of the electric current.

Magnetic Reluctance and Permeability.—The opposition offered by a magnetic circuit to the formation in it of magnetic lines is called *magnetic reluctance*. Compare electrical resistance. The reluctance of any part of a magnetic circuit, like the resistance of any part of an electric circuit, is directly as its length and inversely as its cross-section. It is also proportional to the specific reluctance of the material. But this specific reluctance, unlike the specific resistance of materials forming electrical conductors, often varies greatly with the amount of flux.

Magnetic *permeability* is the reciprocal of reluctance. It is akin to electrical *conductivity*.

Carrying the analogy of the magnetic to the electric circuit still farther, the magnetism developed is often spoken of as a flow of magnetism around the circuit, or as a *magnetic flux*. The cause of magnetic flux is called *magnetomotive force*.

The Electromagnet.—An electromagnet consists of a mass of iron wound with an electric conductor forming a coil or solenoid. When a current flows through this coil, a magnetomotive force is developed which can be shown to be equal to $\frac{4\pi nc}{10}$, where n is the number of turns in the coil and c is the current in amperes. The product nc is called the *ampere-turns*

of the coil. The magnetic flux produced by this magnetomotive force is

$$N = \frac{4\pi nc}{10P},$$

where P is the reluctance of the magnetic circuit. The reluctance is derived from the dimensions and permeabilities of the different parts of the circuit. If a given part of the circuit has a length l, a cross-section A, and a permeability μ,—that is, a specific reluctance $\frac{1}{\mu}$,—the reluctance of that part is $\frac{l}{A\mu}$. μ for air is unity; for iron μ varies from 3000 down to unity, depending upon the density of the magnetic flux through it. μ varies greatly also in different qualities of iron. It is, therefore, often necessary to determine its values for the particular quality of iron it is proposed to use in an electromagnet.

Measurements of Permeability.—To find the permeability of a specimen of iron, it is necessary to determine the magnetic flux corresponding to various magnetomotive forces. This is best done by determining the E.M.F. induced in a small test-coil surrounding the test-piece, when this test-piece is subjected to the influence of a suddenly applied or suddenly reversed magnetomotive force.

The test-piece may be made in the form of a ring which is uniformly wrapped with wire, through which an electric current can be sent to supply the magneto-

motive force. The small test-coil is formed by wrapping over a small portion of the ring a number of turns of fine wire. This is connected to the coil of a ballistic galvanometer. If now a current be sent through the magnetizing coil, magnetism is induced in the iron ring. This is equivalent to suddenly introducing a magnet into the test-coil, and induces in this a current which, acting for an instant upon the needle of the ballistic galvanometer, causes this to swing over a certain angle. The angle of this first swing is called the "throw" of the needle. It can be shown that, when the current is of short duration compared to the time of vibration of the needle, the quantity of electricity passing through the galvanometer coil is proportional to the sine of half the throw. The quantity of electricity flowing is

$$Q = \frac{Et}{R},$$

where E is the E.M.F. developed in the test-coil, t the time it acts, and R the resistance of the test-coil circuit. But E is proportional to the rate of change of magnetic lines, and to the number of turns in the test-coil, or

$$E = \frac{N_1 n_1}{t \times 10^8}, \quad . \quad . \quad . \quad . \quad . \quad (40)$$

where N_1 is the total change in magnetic flux, n_1 the

number of turns in the test-coil, and t the time occupied in making the change. Hence

$$Q = \frac{N_1 n_1}{10^8 R}.$$

It is usual in such measurements to reverse the current in the magnetizing coil. If N represent the total flux produced by the current in one direction, the total change due to reverse is $N_1 = 2N$, and

$$Q = \frac{2Nn_1}{10R^8}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

If K be the constant of the galvanometer, and θ the throw, we have

$$Q = K \sin \tfrac{1}{2}\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

Substituting in (41) and solving for N,

$$N = \frac{10^8 RK \sin \frac{1}{2}\theta}{2n_1}. \quad . \quad . \quad . \quad . \quad . \quad (43)$$

If n be the number of turns in the magnetizing coil and c the current in amperes, the magnetomotive force is

$$M = \frac{4\pi nc}{10}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

If l be the length of the ring, that is, the mean circumference, and A the area of its cross-section, the reluctance is

$$P = \frac{l}{A\mu}.$$

The total flux is then

$$N = \frac{M}{P} = \frac{4\pi n c A \mu}{10 l}.$$

Hence

$$\mu = \frac{N}{A} \div \frac{4\pi n c}{10 l}. \quad . \quad . \quad . \quad . \quad . \qquad (45)$$

But $\frac{N}{A}$ is the magnetic flux per unit area, and is generally represented by the capital letter B. $\frac{4\pi n c}{10 l}$ is the magnetomotive force per unit length, and is represented by H. Hence

$$\mu = \frac{B}{H}, \quad . \quad . \quad . \quad . \quad . \quad . \qquad (46)$$

and finally

$$\mu = \frac{10^8 RK \sin \frac{1}{2}\theta}{2n_1 A} \times \frac{10 l}{4\pi n c}. \quad . \quad . \quad . \qquad (47)$$

The only quantity in the above formula whose determination presents any difficulty is K, the constant of the ballistic galvanometer. This is best determined by experiment. Connect in circuit with the galvanometer a coil of known number of turns, turn that coil through 180° in a known magnetic field, and note the deflection, θ. For example, let the coil lie flat upon a table where the vertical component of the earth's magnetic field is known. Pick up the coil and turn it quickly over. If V represent the vertical

intensity of the earth's field, and A the area of the coil, the number of lines passing through the coil is AV, and the total change of lines in turning the coil over is $2AV$. In formula (43), for N substitute the value of AV; for n_1 substitute the number of turns in the coil used; for R substitute the resistance of the circuit including that coil and the galvanometer; for θ substitute the observed deflection; and solve for K.

The ring is not a convenient form for the test-piece for practical tests, as it is impossible to readily substitute one sample of iron for another. For practical tests a massive soft-iron forging in the form of a U is wrapped with the magnetizing coil. The test-piece, which is a straight round bar of much smaller cross-section, is clamped across the top of the U. Upon this test-piece the small test-coil is placed.

Different specimens of iron may be readily compared by preparing straight round bars of the same dimensions and observing the effect of these upon a magnetic needle placed in a fixed position when the test-pieces are in turn subjected to the influence of a magnetizing coil. This is called the magnetometer method. It is not adapted to exact determinations.

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